An Introductory Review of Ephemerides for Lunar Laser Ranging [and Discussion]

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An introductory review of ephemerides for lunar laser ranging

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Precise predictions of the ranges of the retroreflectors on the Moon from the observing stations on the Earth are required to facilitate the making of observations and also to provide a sound basis for the analysis of the observations. The precision of observations is already such that the theories of the Moon's motion and libration currently used for the ephemerides in the *Astronomical Ephemeris* are inadequate for the analysis, and so the orbital data are generated by numerical integration. New laser systems will give a further improvement in precision, and further factors will have to be taken into account in the predictions. The exploitation of the data will require the development of new analytical theories, but the results will be of value in many different fields of study.

1. Introduction

In general, astronomical ephemerides are required for two different purposes: firstly, as predictions, to facilitate the making of observations; and, secondly, to provide a basis for the reduction, analysis and interpretation of the observations. For the latter purpose it is necessary that the ephemeris represents the physical model on which it is based to a precision that is better than that of the observations. The residual differences between the reduced observations and the ephemeris will then be due to the errors in the adopted parameters of the model and to the deficiencies of the model, such as the omission of significant effects or the use of an inadequate theory. These residuals may then be analysed by, for example, the method of least-squares to improve the parameters and to provide final residuals. For this purpose, it is desirable that the accuracy of the ephemeris be such that the equations of condition in a least-squares solution are linear in the corrections to the parameters. In the development of the ephemeris it is necessary to remember that auxiliary quantities, such as partial derivatives, will be required during the reduction and analysis of the observations; it may be desirable to compute some, or all, of these auxiliary quantities during the computation of the observable quantities. If the final residuals are found to be non-random it is then necessary to look again for sources of systematic error in the observational technique and of deficiency in the model.

In lunar laser ranging we measure the time-interval for light to travel from an emitter on the Earth's surface to a retroreflector on the Moon's surface and back to a receiver, which may not be attached to the same telescope as the emitter. This time-interval is about 2.5 s, since the distance between Earth and Moon is about 3.8×10^8 m. The outgoing laser pulse has a divergence of a few seconds of arc and is currently 5–20 ns long. Because of atmospheric effects the divergence cannot be less than about $1''$, corresponding to the illumination of a region of about 2 km in diameter on the Moon's surface. After reflexion the divergence is greater, and the intensity of the returning pulse is so low that the interval-timer is stopped by the detection of a single returning photon. In order to reduce the number of spurious timings the equipment only responds to a photon that has a wavelength within a certain range, that is travelling in the right direction, and that is detected at the expected time after the emission to within a few microseconds.

The prediction ephemeris used during an observing run must therefore give the direction from the station to the reflector to an accuracy of about 1″ and the range, expressed as a travel time, to about 1 μs. The current systems are manually guided so that the direction is usually given as an offset from a visible, distinct feacture; the offset may be quite large since a retroreflector may be used even if it is not in the illuminated part of the disk. The required direction and range are computed by an on-line minicomputer from a set of coefficients for the observing period. Some of these coefficients may not be known sufficiently accurately at the beginning of the run to permit the immediate acquisition of each reflector, and so some searching may be necessary. Once valid returns are obtained it is possible to apply empirical corrections during the computations for the rest of the run.

The current precision of measurement of the travel time is a few nanoseconds, but it is expected that the precision will be of the order of 0.2 ns when short-pulse lasers come into use. This corresponds to the measurement of the distance between the Earth and the Moon to a precision of about 3 cm. It is therefore desirable that the ephemerides to be used in the analysis should be computed to a precision of about 1 cm, or about 0.1 ns. The ephemerides currently published in the *Astronomical Ephemeris* are quite inadequate for this purpose, although their accuracy corresponds roughly to the accuracy required for the initial predictions.

In the following sections we consider in turn each of the principal contributions to a precise ephemeris for lunar laser ranging. Many of these points will be discussed in more detail in the papers presented later in the meeting, but it is hoped that this review will provide a helpful introduction to these papers. Much of this material is based on the published papers and reports of the members of the LURE team. A general review of the project has been given by Bender, Currie *et al.* (1973).

2. Orbit of the Moon

The accurate computation of the motion of the Moon around the Earth has always represented a challenge to science and mathematics. Until the development of radar and laser ranging the observations were of two principal kinds – firstly, meridian-transit observations of time (and hence right ascension) and of zenith distance (and hence of declination) of the limbs (apparent edge) of the Moon, and, secondly, timings of the occultations of stars (and of the Sun and planets) by the Moon. Both techniques are limited by uncertainties associated with the irregularities of the limb of the Moon, with the errors in star catalogues, and with the timing of the event. The lunar ephemeris currently published in the *Astronomical Ephemeris* is based on the theory developed by E. W. Brown, and modified subsequently to take into account: (*a*) the introduction of ephemeris time, instead of universal time, as the argument of the theory; (*b*) the change to the I.A.U. (1964) System of Astronomical Constants; and (*c*) Eckert's redevelopment of the main problem of the perturbations by the Sun. The coefficients of the individual terms and the published ephemeris are given to a nominal precision of 0.01″ (20 m) in longitude and latitude and of 0.001″ in parallax. After fitting the theory to the observations it is possible to reduce the standard deviation of the residuals of recent occultation observations to about 0.5″ (1 km) of which about one-third is probably due to the uncertainty in the orbital position of the Moon arising from the incompleteness of the development of the planetary perturbations. It was found from radar observations that the uncertainty in the distance was about 0.5 km.

In order to provide quickly an ephemeris that is sufficiently precise for use with lunar laser ranging observations, it has been necessary to generate the orbital positions by numerical integration of the equations of motion, taking into account all known perturbations and differences from basic Newtonian mechanics. The principal perturbations are due to the Sun but the following effects must also be taken into account: the perturbations by the planets, the effects of the non-sphericity of the Earth and the Moon, relativistic effects, and tidal forces. The analysis of the laser ranging data will lead to: (*a*) significant improvements in the determination of the elements of the orbits of the Moon around the Earth, and of their centre of mass around the Sun; (*b*) tests of theories of relativity since the total relativistic effect of 1.2 m is much greater than the precision of the observations; and (*c*) a better knowledge of the effects of tidal reactions on the orbit. The classical techniques for the observation of the position of the Moon have led to estimates of the secular (tidal) acceleration in longitude, but there have been considerable discordances between the determinations. The tidal acceleration corresponds to a recession of the Moon from the Earth by about 3 cm per year, but it appears that there are also periodic perturbations which will be detectable by laser ranging. A model of the tidal bulge raised by the Moon on the Earth can be adopted, and the forces on the Moon due to this bulge can be calculated and integrated numerically. Improvements in parameters defining the bulge can then be obtained from the analysis of the range residuals.

The partial derivatives required for use in the equations of condition may be determined numerically by repeating the numerical integration many times with small changes in the starting values or in the parameters defining the physical model that is represented by the integration. Alternatively, they may be calculated from analytic expressions derived from an analytical theory that is believed to correspond to the model being integrated. These two methods provide useful checks on each other. The technique of numerical integration is quicker and more economical in the short-term, but it has the disadvantage that the effects of rounding errors in the computation may accumulate very rapidly if the integration is continued for many revolutions. This makes it difficult to study the long-term changes in the orbit, such as those that would correspond to a change in the constant of gravitation with time. For an analytical theory the precision of the computation (although not the accuracy of the ephemeris) is independent of the date for which the theory is evaluated. The development of an analytical theory of the motion that is better than Brown's theory by a factor of 10 000 is a formidable task, but work is in hand, especially in France. Even when the new theory is completed much effort will be required to compare it with the numerical integrations and to use it in the analysis of the data.

3. The position of the station

It is necessary to compute the position of the observing station relative to the inertial frame of reference used for the orbit of the Moon. This computation consists essentially of two parts – the position of the station relative to a reference frame fixed in the Earth, and the position of this frame relative to the inertial frame. Since the Earth is not a rigid body it is by no means obvious how the reference frame for the Earth should be defined, although its origin is required to be coincident with the centre of mass of the Earth; one axis should correspond closely to the axis of figure.

Even the specification of the position with respect to the fixed frame is not a trivial matter since the amplitudes of the tidal motions are of the order of 30 cm, and so must be taken into account. It may be desirable to model also the possible local and continental drifts of the station, which may be of the order of 3 cm per year, rather than to leave the total effect in the final residuals. The mean positions of each station with respect to the adopted reference frame will be determined from the observations, and it will be necessary to use other methods to relate this frame to the conventional geodetic reference system. Mobile stations will be required if the full potential of the method is to be exploited for geodesy and the study of plate tectonics.

The transformation from the frame fixed in the Earth to the inertial frame consists of three main steps: (*a*) the transformation for polar motion, i.e. between the adopted axis of figure and the axis of rotation; (*b*) the transformation for precession and nutation, i.e. between the axis of rotation and the axis of the inertial frame; and (*c*) the transformation corresponding to the non-uniform rotation of the Earth around its axis of rotation. Of these three steps only the effect of precession and nutation can be computed in advance, and even here the theory on which the published ephemerides are based is inadequate for lunar laser ranging; the theory is based on a rigid model for the Earth, but it will be necessary to include the effects of the elasticity of the mantle and the fluidity of the core in the new model. It seems likely that lunar laser ranging will provide improved determinations of such fundamental data as the constants of luni-solar precession and the obliquity of ecliptic, as well as the parameters for the Earth model used in the theory of nutation.

The sensitivity of the range to the motion of the station around the axis of the Earth is such that it is already possible to determine the difference between universal time and atomic time to a precision that is comparable with that given by the classical astronomical methods. The polar motion is also detectable but data from at least two stations (separated by about 90° in longitude) are required. A time-difference of 0.001 s corresponds to a change of position at latitude 30° of 40 cm, and a polar motion of 0.01″ corresponds to a change of position of 30 cm, so that eventually laser ranging should lead to an improvement over present techniques by a factor of about 10 in precision. Laser ranging suffers from the disadvantages that observations cannot be made for a few days around each new moon and that, unless there is a good distribution of stations in longitude and between the northern and southern hemispheres, there may be other periods when the data will be poor or even non-existent. Hence, it seems probable that, although laser ranging will contribute to the rapid determination of data on the Earth's rotation, it will be better to use definitive data obtained by other methods in the full analysis of the ranging data, thus reducing the noise and improving the determination of other parameters that are best determined from lunar laser ranging.

4. The positions of the retroreflectors

The positions of the retroreflectors relative to each other and to some adopted reference frame in the Moon cannot be determined with the same precision as positions of stations on the Earth, since the changes in the orientations of their selenocentric position vectors with respect to the line of sight are at most 8°, compared with changes of up to 60° for the geocentric position vectors of the observing stations. Even so, the analysis will demand a considerable increase in the precision, and hence complexity, of the theory of the rotation of the Moon, or lunar librations. The nominal precision of the ephemerics of the physical libration of the Moon given in the *Astronomical Ephemeris* is 0.01°, corresponding to an error in position on the surface of 300 m. This precision is adequate for directional observations from the Earth, but the required precision in angle will need to be 0.1″ if the ephemeris is to represent the physical model to a precision of 1 m in the positions of the retroreflectors in space.

The rotational ephemeris that is now used for the analysis of the ranging data is based on numerical integrations, but it seems likely that analytical theories of adequate precision will become available much earlier than will be the case for the orbit of the Moon. The new theories will require a knowledge of the coefficients of the gravitational potential for the Moon (and hence for its distribution of mass) to at least the third order. Some of these coefficients may be available from analysis of other data, such as the motions of lunar orbiters, but others will be best determined from the laser ranging data, thus leading to further improvements in our knowledge and structure of the Moon. It is not yet known whether there is a 'free nutation' of the Moon, corresponding to the polar motion of the Earth. This seems unlikely but if one is found to be present it would provide yet another indirect way of studying the internal properties of the Moon.

The positions of the retroreflectors can be combined with corresponding data on the positions of the radio transmitters associated with other experimental packages on the lunar surface to provide a precise basis for a coordinate system on the surface of the nearside of the Moon. The positions of the radio transmitters can be directly related to the positions of astronomical radio sources using the techniques of very long base-line radio interferometry, thus providing a precise link between the inertial frame defined by the Sun–Earth–Moon system and the frame defined by extragalactic radio sources. The dynamical frame can also be related to the stellar reference frame through occultation observations, but the uncertainties in our knowledge of the topography of the marginal zone of the Moon are likely to limit the accuracy that can be achieved.

5. Other factors

Apart from the limitation imposed by the instrumental difficulties of generating very short pulses and of detecting and timing the returning photons, the technique is limited by other uncertainties in the modelling of the system. Perhaps the most important is the uncertainty in the computations of the delays that occur in the Earth's atmosphere. At the zenith the total delay, compared with transmission *in vacuo*, is equivalent to about 2 m in path-length, and it is claimed that the delay can be computed to a precision of better than 1 cm; the effect depends primarily on the surface pressure. As the zenith distance increases so the delay, and the uncertainties increase – one of the reasons why a station in the southern hemisphere will be particularly valuable is that it will be able to observe the Moon near the zenith when it is at its southernmost declination. Another factor that leads to the conclusion that the technique will never achieve a precision of 1 cm is that the effective point of reflexion of the signal cannot be modelled to better than this. It would also be difficult to calibrate the emission and detection system to this precision.

6. Conclusions

The task of generating and using ephemerides that will be of adequate precision and accuracy to enable us to exploit the full potential of lunar laser ranging observations is one that will require the cooperative efforts of many scientists. These efforts promise, however, to lead to new results in a wide variety of fields – celestial mechanics, relativity, evolution of the Earth–Moon system, fundamental astrometry, determination of universal time and polar motion, geodesy, plate tectonics, selenodesy and the structure of the Moon – and to justify the technological efforts that have been required to place the retroreflectors on the Moon and to develop laser and timing systems of such high precision.

Reference (Wilkins)

Bender, P. L., Currie, D. G. *et al.* 1973 The lunar laser ranging experiment. *Science, N.Y.* **182**, 229–238.

Discussion

J. A. Weightman (*Geodetic Office, Elmwood Avenue, Feltham, Middlesex*). If the entire return from a lunar laser ranger signal is often only one photon how can one be sure that it is one's own photon and not a completely spurious signal? To go to more sophisticated theory so as to predict even more precisely when the one photon could be expected would seem only to increase the probability that the observations would confirm the theory.

G. A. Wilkins. The incoming photons are finely filtered in wavelength and direction, but the timing gate is sufficiently long with respect to the timing precision to allow the timings during a period of observation to be analysed in order to determine the time-interval for which the number of timings is significantly greater than the number expected for the background noise.